Fisher's exact test

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Fisher's exact test

On the basis of the hypergeometric distribution, Fisher's exact test is used to test the independence between two qualitative variables in a contingency table. E.g., we can use Fisher's exact test to test if women and men have identical passing/not passing proportions after an examination:

Sex $\downarrow$ / Result $\rightarrow$	Passing	Not passing	Total
Man	8	5	13
Woman	3	12	15
Total	11	17	28

Fisher's exact test

Under the null hypothesis, two variables are independent, that is, there is no difference between men and women about passing/not passing proportions. The expected frequencies under independence will be those in the following table, multiplying marginal frequencies for each cell, and dividing by the sampling size (e.g., $(11 \times 13)/28 = 5.1$):

Sex $\downarrow$ / Result $\rightarrow$	Passing	Not passing	Total
Man	5.1	7.9	13
Woman	5.9	9.1	15
Total	11	17	28

Fisher's exact test

Comparing empirical (real) and expected frequencies, it seems that men pass more frequently, and so that is the alternative hypothesis when we reject the null hypothesis. So, "the strange thing" is on the upper side: comparing empiric and theoretical frequencies, it semms that men tend to pass more frequently, so that's the alternative hypothesis, when we reject the null hypothesis. So, in that cell "the strange thing" is on the upper side: 8, 9, 10 or 11 men passing. We have to calculate the probability of that.
Under the null hypothesis we have independence, so 11 passing persons among 28 persons distribute randomly among men and women; or, on the other side, 13 men among 20 persons distribute randomly among passing and not passing persons.

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So the probability of being 8, 9, 10 or 11 passing men may be calculated in any these two ways:

- $$P[8, 9, 10, 11] = \frac{\binom{11}{8}\binom{17}{5}}{\binom{28}{13}} + \frac{\binom{11}{9}\binom{17}{4}}{\binom{28}{13}} + \frac{\binom{11}{10}\binom{17}{3}}{\binom{28}{13}} + \frac{\binom{11}{11}\binom{17}{2}}{\binom{28}{13}} = 0.0309$$

- $$P[8, 9, 10, 11] = \frac{\binom{13}{8}\binom{15}{3}}{\binom{28}{11}} + \frac{\binom{13}{9}\binom{15}{2}}{\binom{28}{11}} + \frac{\binom{13}{10}\binom{15}{1}}{\binom{28}{11}} + \frac{\binom{13}{11}\binom{15}{0}}{\binom{28}{11}} = 0.0309$$

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If significance level is %5 , we must reject the null hypothesis, because p-value (0.0309) is smaller, and so we conclude that sex and exam results are related to each other.
We should note that we are performing an one-sided test, because alternatively we are accepting that men pass more frequently. So we have to compare p-value with α, and not with $\alpha/2$.

Fisher's exact test

Should we take always as a pivot for the calculations the north-west frequency? The answer is no. We can take any frequency as a pivot, they are perfectly interchangeable and the p-value will be always the same, because of the symmetries of the hypergeometric distribution. But it's always better to take the smallest frequency to simplify the calculations.

Fisher's exact test

Why is Fisher's test an exact test? Because it calculates exactly the p-value. Other tests for independence (e.g., chi-square test) are approximations, as they assume that frequencies are continuous, but they are not. So Fisher's exact test is specially useful when frequencies are small, so we can't at all assume that they are continuous.

Fisher's exact test

Fisher's exact test has some disadvantages:

- Marginal frequencies are fixed, and that's a very strict condition (why must we take internal frequencies as variable, and marginal frequencies as fixed?).
- It's a conservative test, as it tends to accept the null hypothesis, specially for small frequencies, because it's difficult to have probabilities smaller than α. So, it's recommended to set bigger α values than usually (e.g., instead of %1, we set %10).